Ez simetrikoa

↓

$\sigma = \{\lambda_1(k_1), \lambda_2(k_2), \ldots, \lambda_n(k_n)\}$

↓

$\dim V(\lambda_i) = d_i$

$\sum d_i = \sum k_i = n$ ⟶ Diagonalgarria

↓

$\exists P$ matrize / $D = P^{-1} \cdot A \cdot P$

↓

Diagonalketa

Simetrikoa

↓

Diagonalgarria

↓

$\exists P_{ogn}$ matrize ortonormala

$D = (P_{ogn})^{t} \cdot A \cdot P_{ogn}$

↓

Diagonalketa ortogonala

$AX - \lambda IX = (0)$

$AX = \lambda X$ / $|A - \lambda I| = 0$ ⟶ **λ autobalioak**

↓

X: autobektoreak

↓

$P = \begin{pmatrix} x_1 \\ x_2 \\ x_n \end{pmatrix}$

$\sigma(A) = \{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n\}$

↓

espektroa

! Anizkoitasun maila kontutan hartu behar da.

$D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_n \end{pmatrix}$

$V \begin{cases} \mathbb{R}^n \; ; \; n \in \mathbb{R} \\ P_n \; ; \; \forall \vec{v} \in P_n \\ M_{m \times n}(\mathbb{R}) \\ C\,[a,b] \end{cases}$

V , $+$, $\circ$ ⟶ Kanpo konposaketa legea

↓

Barne konposaketa legea

⟶ $\{(1,1,0), (3,2,0), (2,3,0) \ldots\}$

Bektoreekin, konbinazio linealen bitartez, itxadura lineala sortu daiteke. Itxadura multzo bat da.

Azpiespazio bektoriala ⟵ $S = \mathcal{L}(F) = \mathcal{L}(B_s)$ ⟶ S-ren **dimentsioa**

$h(B_s) = h = \dim S$

↓ sortzaile lotua ⟷ ↓ sortzaile askea

bektore linealki independenteen kopurua ⟵ Heina $\begin{cases} h = m \text{ askea} \\ h < m \text{ lotua} \end{cases}$

m bektore kopurua

Oinarri baten laguntzaz: bektoreak sortzeko behar diren heina kalkulatu

$\dim S < \dim V$

↳ V-ren azpiespazio propioa da

$\dim S = \dim V$

↳ $S = V$

$\dim \geq 1$

↳ Dimentsioa beti izango da handiago edo berdin bat

$\|\bar{x}-\bar{y}\| = \partial(\bar{x},\bar{y}) = +\sqrt{\langle(\bar{x}-\bar{y}),(\bar{x}-\bar{y})\rangle}$

$\bar{x} = (1,2,3)$
$\bar{y} = (2,1,4)$ $\rightarrow \bar{x}-\bar{y} = (-1,1,-1)$

$\partial(\bar{x},\bar{y}) = \sqrt{(-1)^2+1^2+(-1)^2} = \sqrt{3}$

B: bektoreen arteko **distantzia**

B: bektoreen arteko **angelua**

$\cos\theta = \dfrac{\langle\bar{x},\bar{y}\rangle}{\|\bar{x}\|\cdot\|\bar{y}\|}$

bektorea

$(V, \langle\ \rangle)$

Bektore baten **norma** euklidearra

$\|\bar{x}\| = +\sqrt{\langle\bar{x},\bar{x}\rangle}$

$\bar{x} = (1,2,3)$

$\|\bar{x}\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$\bar{y} = (2,1,4)$

$\|\bar{y}\| = \sqrt{2^2+1^2+4^2} = \sqrt{21}$

biderketa eskalarra

Bektore baten **proiekzioa** beste bektore baten gainean → ~~B: bektoreen arteko angelua~~ ~~$\cos\theta =$~~

$P_{\bar{y}}\bar{x} = \dfrac{\langle\bar{x},\bar{y}\rangle}{\|\bar{y}\|^2}\cdot\bar{y}$

$P_{\bar{x}}\bar{y} = \dfrac{\langle\bar{y},\bar{x}\rangle}{\|\bar{x}\|^2}\bar{x}$

$P_{\bar{y}}\bar{x} = \dfrac{16}{21}(2,1,4) = \left(\frac{32}{21},\frac{16}{21},\frac{64}{21}\right)$

$P_{\bar{x}}\bar{y} = \dfrac{16}{14}(1,2,3) = \left(\frac{8}{7},\frac{16}{7},\frac{24}{7}\right)$

Oinarri baten ortogonalketa

Hurbilketa lineal hobea

Gram-Schmidt prozedura

$B_S = (\bar{u}_1,\bar{u}_2,\ldots\bar{u}_n)$

$B_{og} = (\bar{v}_1,\bar{v}_2,\ldots,\bar{v}_n)$

$\mathcal{L}(B_S) = S$

$\mathcal{L}(B_{og}) = S$

$\bar{v}_1 = \bar{v}_1 \quad \bar{v}_2 = \bar{u}_2 - \dfrac{\langle\bar{u}_2,\bar{v}_1\rangle}{\|v_1\|^2}\cdot\bar{v}_1$

$\bar{v}_3 = \bar{u}_3 - \dfrac{\langle\bar{u}_3,\bar{v}_1\rangle}{\|v_1\|^2}v_1 - \dfrac{\langle\bar{u}_3,\bar{v}_2\rangle}{\|v_2\|^2}v_2$

$\langle v_1,v_2\rangle = 0$
$\langle v_1,v_3\rangle = 0$
$\langle v_2,v_3\rangle = 0$

$v_n = u_n - P_{\bar{v}_1}\bar{u}_n - P_{\bar{v}_2}\bar{u}_n - P_{v_{n-1}}\bar{u}_n$

Bektoreak binaka hartuta, elkarren arteko biderkadura eskalarrak zero izango dira

ELS BATERAEZINA **SOLUZIO HURBILDUA**

1) B_S → oinarri bat lortu

2) B_S → B_{og} (oinarria ortogonaldu)

3) $\bar{b}$-ren $HL(S) = \bar{b}'$

4) $A\bar{x} = \bar{b}'$ → Bateragarria

5) Sistema ebatzi, $\bar{x}$: sol hurbildua

Bektoreen moduloa 1 dela ziurtatzeko

Ortonormalizazioa

$\left\{\dfrac{\bar{v}_1}{\|v_1\|},\dfrac{\bar{v}_2}{\|v_2\|},\dfrac{\bar{v}_3}{\|v_3\|}\ldots\right\}$

DIAGONALKETA

1) $|A-\lambda I| = 0$

2) Autobaloreak → espektroa → D

Autobektoreak → P

3) P^{-1}, P_{og}, $(P_{og})^t$...

ELS BATERAEZINA SOLUZIO HURBILDUA

$$x_1 + x_3 = 1$$
$$x_2 + 2x_3 = 1$$
$$x_1 + 2x_2 + 2x_3 = 1$$
$$x_1 + x_2 + 3x_3 = 1$$

$$A_i = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 3 & 1 \end{pmatrix}$$
$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ \bar{u}_1 & \bar{u}_2 & \bar{u}_3 & \bar{b} \end{array}$$

$h(A) \neq h(A_i)$

$B_3 = \{(1,0,1,1),(0,1,2,1),(1,2,2,3)\}$

$h(B_3) = 3$: bektore kopurua

$\downarrow$

Oinarria da

$B_{og} = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$

$\|v_1\|^2 = 3 \qquad \langle \bar{u}_2, \bar{v}_1\rangle = 3$

$\|v_2\|^2 = 3 \qquad \langle \bar{u}_3, \bar{v}_1\rangle = 6$

$\langle \bar{u}_3, \bar{v}_2\rangle = 3$

$$\bar{v}_1 = \bar{u}_1 = (1,0,1,1)$$

$$\bar{v}_2 = \bar{u}_2 - \frac{\langle \bar{u}_2, \bar{v}_1\rangle}{\|v_1\|^2} v_1 = (0,1,2,1) - \frac{3}{3}(1,0,1,1) = (-1,1,1,0)$$

$$\bar{v}_3 = \bar{u}_3 - \frac{\langle \bar{u}_3, \bar{v}_1\rangle}{\|v_1\|^2} v_1 - \frac{\langle \bar{u}_3, \bar{v}_2\rangle}{\|v_2\|^2} v_2 = (1,2,2,3) - \frac{6}{3}(1,0,1,1) - \frac{3}{3}(0,1,2,1) = (0,1,-1,1)$$

$$B_{og} = \{(1,0,1,1),(-1,1,1,0),(0,1,-1,1)\}$$

$$\bar{b}' = \sum_{i=1} P_{\bar{v}_i}\bar{b} = \frac{\langle \bar{b}, \bar{v}_1\rangle}{\|\bar{v}_1\|^2}\bar{v}_1 + \frac{\langle \bar{b}, \bar{v}_2\rangle}{\|v_2\|^2}\bar{v}_2 + \frac{\langle \bar{b}, \bar{v}_3\rangle}{\|v_3\|^2}\bar{v}_3 = \frac{3}{3}\bar{v}_1 + \frac{1}{3}\bar{v}_2 + \frac{1}{3}\bar{v}_3 = (2/3, 2/3, 1, 4/3)$$

$\langle \bar{b}, \bar{v}_1\rangle = 3 \qquad \|v_1\|^2 = 3$

$\langle \bar{b}, \bar{v}_2\rangle = 1 \qquad \|v_2\|^2 = 3$

$\langle \bar{b}, \bar{v}_3\rangle = 1 \qquad \|v_3\|^2 = 3$

$$A\bar{x} = \bar{b} \Rightarrow A\bar{x} = \bar{b}'$$

$$A_i' = \begin{pmatrix} 1 & 0 & 1 & 2/3 \\ 0 & 1 & 2 & 4/3 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 3 & 4/3 \end{pmatrix} \underset{E_4 - E_1}{\overset{E_3 - E_1}{\sim}} \begin{pmatrix} 1 & 0 & 1 & 2/3 \\ 0 & 1 & 2 & 4/3 \\ 0 & 2 & 1 & 1/3 \\ 0 & 1 & 2 & 4/3 \end{pmatrix} \underset{E_4 - E_2}{\overset{E_3 - E_2}{\sim}} \begin{pmatrix} 1 & 0 & 1 & 2/3 \\ 0 & 1 & 2 & 4/3 \\ 0 & 0 & 3 & -3/3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 4/3 \\ 0 & 1 & 2 & 4/3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$h(A) = h(A_i) = 3$

$x_1 = -1/3 \qquad x_2 = -4/3 \qquad x_3\ 1$

$$e = d(\bar{b}, \bar{b}') = \|\bar{b} - \bar{b}'\| = \sqrt{(1/3)^2 + (1/3)^2 + (-1/3)^2} = \sqrt{1/3}$$

DIAGONALIZA

$$B = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} \quad |B - \lambda I| = 0 \quad \begin{vmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{vmatrix} = (2-\lambda)(-2-\lambda)+3 = 0 \Rightarrow -4 - \cancel{2\lambda} + \cancel{2\lambda} + \lambda^2 + 3 \Rightarrow \lambda^2 = 1$$

$$\lambda_1 = 1 \ (k=1) \qquad \sigma(B) = \{1, -1\} \qquad D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$\lambda_2 = -1 \ (k=1)$$

$\boxed{\lambda = 1}$

$$\begin{pmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{pmatrix} \Rightarrow \begin{pmatrix} 2-1 & -3 \\ 1 & -2-1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -3 \\ 1 & -3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -3 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & -3 & 0 \\ 1 & -3 & 0 \end{array}\right) \quad x_1 - 3x_2 = 0 \rightarrow x_1 = 3x_2 \rightarrow (x_1, x_2) = (3x_2, x_2) = x_2(3, 1)$$

$$V(1) = \mathcal{L}\{(3, 1)\}$$

$\boxed{\lambda = -1}$

$$\begin{pmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{pmatrix} \Rightarrow \begin{pmatrix} 2+1 & -3 \\ 1 & -2+1 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & -3 \\ 1 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 3 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 3 & -3 & 0 \\ 1 & -1 & 0 \end{array}\right) \quad x_1 \quad x_2 \rightarrow (x_1, x_2) = (x_2, x_2) = x_2(1, 1)$$

$$V(-1) = \mathcal{L}\{(1, 1)\}$$

$$P = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$$

$$D = P^{-1} \cdot B \ P \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$$